Section 7.5

Conditional Probability: If A and B are events with $P(B) \neq 0$, then the probability of A given B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Principle for Conditional Probability: If *A* and *B* are events, then $P(A \cap B) = P(A \mid B)P(B)$.

Independent Events: The events are independent if

$$P(A \cap B) = P(A)P(B).$$

Equivalent formulas (assuming neither A nor B is impossible) are

$$P(A \mid B) = P(A)$$

and
$$P(B \mid A) = P(B).$$

If two events A and B are not independent, then they are **dependent**. The property $P(A \cap B) = P(A)P(B)$ can be extended to three or more independent events (that is, each of them is independent of each of the other two and of their intersection), then, among other things,

$$P(A \cap B \cap C) = P(A)P(B)P(C).$$

Testing for Independence: To check whether two events *A* and *B* are independent, we compute P(A), P(B), and $P(A \cap B)$. If $P(A \cap B) = P(A)P(B)$, the events are independent.

Problem 1. Compute the indicated quantity.

- a) $P(B) = .6, P(A \cap B) = .3$. Find $P(A \mid B)$.
- b) $P(A | B) = .4, P(A \cap B) = .1$. Find P(B).

c) P(A) = .2, P(B) = .2. A and B are independent. Find $P(A \cap B)$.

Solutions here.

Problem 2. Supply the missing quantities.



Solution here.

Problem 3. Find the conditional probabilities of the indicated events when two fair dice (one red and one green) are rolled.

- a) The sum is 6, given that the green one is either 4 or 3.
- b) The red one is 4, given that the green one is 4.

Solutions here.

Problem 4. A bag contains three red marbles, two green ones, one fluorescent pink one, two yellow ones, and two orange ones. Suzan grabs four at random. Find the probabilities of the indicated events.

- a) She gets all the red ones, given that she does not get the fluorescent pink one.
- b) She gets one of each color other than fluorescent pink, given that she gets the fluorescent pink one.
- c) She gets at least two red ones, given that she gets at least one green one.

Solutions here.